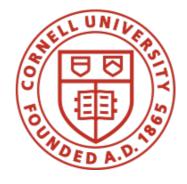


Nate Foster Cornell University





Nate Foster



Justin Hsu



David Kahn



Dexter Kozen



Praveen Kumar

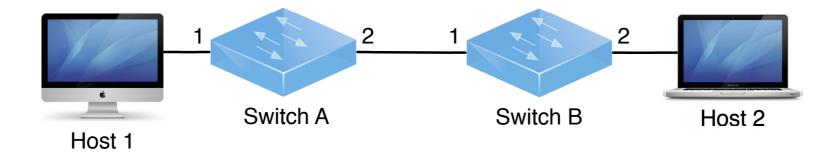


Alexandra Silva

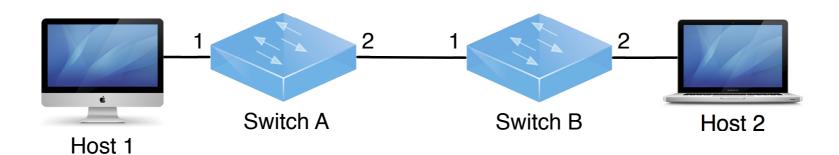


Steffen Smolka

Network Verification



Network Verification

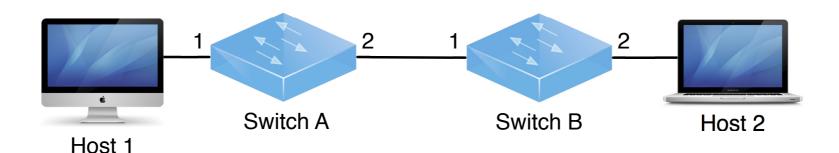




"Are packets routed between hosts?"

"Are ssh packets dropped?"

Network Verification





"Are packets routed between hosts?"

"Are ssh packets dropped?"

Verification Tool



Inputs: Network config & topology + question

Outputs: "Yes" / "No" + counterexample

State of the art tools verify reachability properties:

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Waypointing: every packet traverses a firewall

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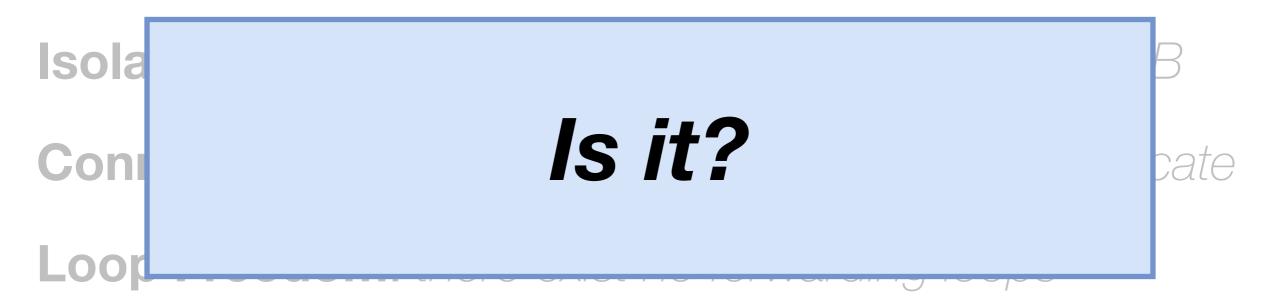
Loop Freedom: there exist no forwarding loops

Access Control: Internet packets cannot enter the VLAN

Key Assumption: network behavior is deterministic

State of the art tools verify reachability properties:

Waypointing: every packet traverses a firewall



Access Control: Internet packets cannot enter the VLAN

Key Assumption: network behavior is deterministic

Probabilistic Network Behavior

It's often reasonable to model networks deterministically

But what if...

- → ... a link or switch fails?
- → ... the network employs resilient routing?
 - ◆ "what's the probability of packet delivery?"
 - "what's the expected path length?"
- ★... the network employs traffic engineering?
 - "what's the expected congestion of this link?"

Probabilistic NetKAT

Probabilistic NetKAT

A DSL for programming, modeling & reasoning about probabilistic networks

Probabilistic NetKAT

Nate Foster^{1(⊠)}, Dexter Kozen¹, Konstantinos Mamouras², Mark Reitblatt¹, and Alexandra Silva³

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Abstract. This paper presents a new language for network programming based on a probabilistic semantics. We extend the NetKATlanguage with new primitives for expressing probabilistic behaviors and enrich the semantics from one based on deterministic functions to one based on measurable functions on sets of packet histories. We establish fundamental properties of the semantics, prove that it is a conservative extension of the deterministic semantics, show that it satisfies a number of natural equations, and develop a notion of approximation. We present case studies that show how the language can be used to model a diverse collection of scenarios drawn from real-world networks.

1 Introduction

Formal specification and verification of networks has become a reality in recent years with the emergence of network-specific programming languages and property-checking tools. Programming languages like Frenetic [11], Pyretic [35], Maple [51], FlowLog [37], and others are enabling programmers to specify the intended behavior of a network in terms of high-level constructs such as Boolean predicates and functions on packets. Verification tools like Header Space Analysis [21], VeriFlow [22], and NetKAT [12] are making it possible to check properties such as connectivity, loop freedom, and traffic isolation automatically.

However, despite many notable advances, these frameworks all have a fundamental limitation: they model network behavior in terms of deterministic packetprocessing functions. This approach works well enough in settings where the network functionality is simple, or where the properties of interest only concern the forwarding paths used to carry traffic. But it does not provide satisfactory accounts of more complicated situations that often arise in practice:

- Congestion: the network operator wishes to calculate the expected degree of congestion on each link given a model of the demands for traffic.
- Failure: the network operator wishes to calculate the probability that packets will be delivered to their destination, given that devices and links fail with a

Cantor Meets Scott: Semantic Foundations for Probabilistic Networks



Comell University, USA

Dexter Kozen Comell University USA

Alexandra Silva University College London, UK

ProbNetKAT is a probabilistic extension of NetKAT with a de ics based on Markov kernels. The language is notational semantics based on Markov kernels. The language is expressive enough to generate continuous distributions, which raises the question of how to compute effectively in the language. This paper gives an new characterization of ProNNetKAT's semantics using domain theory, which provides the foundation needed to build a practical implementation. We show how to use the semantics to approximate the behavior of arbitrary ProNnetKAT programs using distributions with finite support. We develop a prototype implementation and show how to use it to solve a variety of problems including characterizing the expected congestion induced by different routing schemes and reasoning probabilistically about reachability in a network.

Categories and Subject Descriptors D.3.1 [Programming Languages]: Formal Definitions and Theory—Semantics

Keywords Software-defined networking, Probabilistic semantics, Kleene algebra with tests, Domain theory, NetKAT.

1. Introduction
The recent energence of software-defined networking (SDN) has led to the development of a number of domain-specific programming languages (Foster et al 2011; Monsanto et al 2013; Wellmy et al [2013; Wellmy et al [2013] Wellmy et al [2014] and rassoning tools (Kazemian et al. 2012; Marndet et al. 2013; Anderson et al [2014; Foster et al [2015] for networks. But there is still a large gap between the models provided by these languages and the realities of modern networks. In particular, most existing SDN languages have semantics based on deterministic packet-processing functions, which makes it impossible to encode probabilistic behaviors. This is unfortunate because in the real world, network operators often use randomized protocols and probabilistic reasoning to achieve good performance.

Previous work on ProbNetKAT (Foster et al. (2016) proposed an extension to the NetKAT language (Anderson et al. (2014; Foster et al. 2015) with a random choice operator that can be used to express a variety of probabilistic behaviors. ProbNetKAT has a compositional semantics based on Markov kernels that conservatively extends the deterministic NetKAT semantics and has been used to reason about various aspects of network performance include used to reason about various aspects of network performance including congestion, fault tolerance, and latency. However, although the language enjoys a number of attractive theoretical properties, there are some major impediments to building a practical implementation: (i) the semantics of iteration is formulated as an infinite process rather than a fixpoint in a suitable order, and (ii) some programs generate continuous distributions. These factors make it difficult to determine when a computation has converged to its final value, and there are also challenges related to representing and analyzing distributions with infinite support.

This paper introduces a new semantics for ProbNetKAT, following the approach pioneered by Saheb-Djahroni, Jones, and Plotkin (Saheb-Djahronii 1980, 1978; Jones 1989; Plotkin 1982; Jones and Plotkin 1989; Whereas the original semantics of ProbNet (Saheb-Djahronii 1980, 1978; Jones 1989; Plotkin 1982).

lowing the approach pioneered by Saheb-Djahromi, Jones, and Plotkin (Saheb-Djahromi 1960, 1978; Jlones; 1989; Plotkin 1982; Jones and Plotkin 1989). Whereas the original semantics of Prob-RelKAT was somewhat imperative in nature, being based on stochastic processes, the semantics introduced in this paper is purely functional. Nevertheless, the two semantics are closely related—we give a precise, technical characterization of the relationship between them. The new semantics provides a suitable foundation for building a practical implementation, it provides new insights into the nature of probabilistic behavior in networks, and it opens up several interesting theoretical questions for future work.

Our new semantics follows the order-theoretic tradition established in previous work on Scott-style domain theory (Scott[1972; Abramsky and Jung 1994). In particular, Scott-continuous maps on algebraic and continuous DCPOs both play a key role in our development. However, there is an interesting twist: NetKAT and ProbNetKAT are not state-based as with most other probabilistic systems, but are rather throughput-based. A ProbNetKAT program can be thought of as a filter that takes an input set of packet histories and generates an output randomly distributed on the measurable space 2° of sets of packet histories. The closest thing to a "state" is a set of packet histories, and the structure of on the measurable space 2° of sets of packet histories. The closest thing to a "state" is a set of packet histories, and the structure of the domains are not flat domains as in traditional domain theory, but are instead the DCPO of sets of packet histories ordered by the subset relation. Another point of departure from prior work is that the structures used

Probabilistic Program Equivalence for NetKAT*

STEFFEN SMOLKA, Cornell University, USA PRAVEEN KUMAR, Cornell University, USA NATE FOSTER, Cornell University, USA IUSTIN HSU, Cornell University, USA DAVID KAHN, Cornell University, USA DEXTER KOZEN, Cornell University, USA ALEXANDRA SILVA, University College London, UK

We tackle the problem of deciding whether two probabilistic programs are equivalent in Probabilistic NetKAT, a formal language for specifying and reasoning about the behavior of packet-switched networks. We show that the problem is decidable for the history-free fragment of the language by developing an effective decision procedure based on stochastic matrices. The main challenge lies in reasoning about iteration, which we address by designing an encoding of the program semantics as a finite-state absorbing Markov chain, whose limiting distribution can be computed exactly. In an extended case study on a real-world data center network, we automatically verify various quantitative properties of interest, including resilience in the presence of failures, by analyzing the Markov chain semantic

Program equivalence is one of the most fundamental problems in Computer Science: given a pair of programs, do they describe the same computation? The problem is undecidable in general, but it can often be solved for domain-specific languages based on restricted computational models. For example, a classical approach for deciding whether a pair of regular expressions denote the same language is to first convert the expressions to deterministic finite automata, which can then be checked for equivalence in almost linear time [32]. In addition to the theoretical motivation, there are also many practical benefits to studying program equivalence. Being able to decide equivalence enables more sophisticated applications, for instance in verified compilation and program synthesis. Less obviously—but arguably more importantly—deciding equivalence typically involves finding some sort of finite, explicit representation of the program semantics. This compact encoding can open the door to reasoning techniques and decision procedures for properties that extend far beyond straightforward program equivalence.

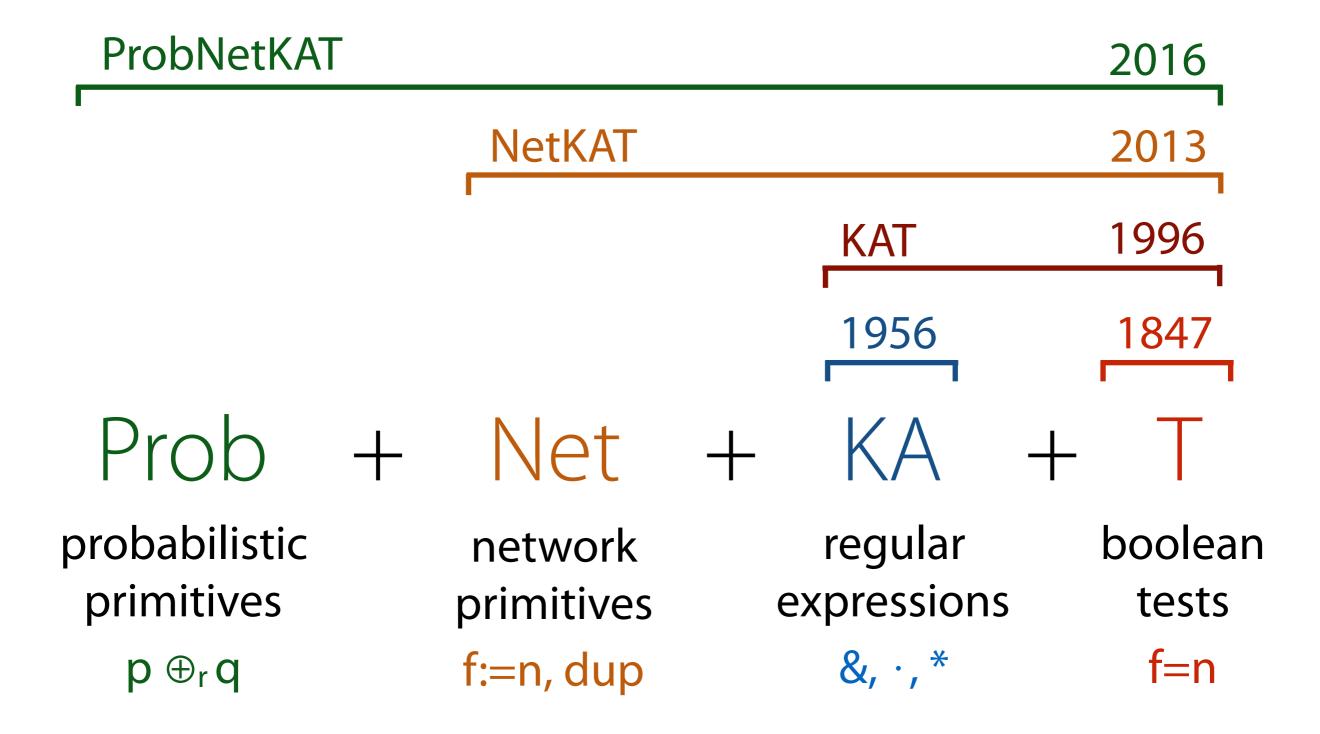
With this motivation in mind, this paper tackles the problem of deciding equivalence in Probabilistic NetKAT (ProbNetKAT), a language for modeling and reasoning about the behavior of packet-switched networks. As its name suggests, ProbNetKAT is based on NetKAT [3, 9, 30], which is in turn based on Kleene algebra with tests (KAT), an algebraic system combining Boolean predicates and regular expressions. ProbNetKAT extends NetKAT with a random choice operator and a semantics based on Markov kernels [31]. The framework can be used to encode and reason about randomized protocols (e.g., a routing scheme that uses random forwarding paths to balance load [33]); describe uncertainty about traffic demands (e.g., the diurnal/nocturnal fluctuation in access patterns commonly seen in networks for large content providers [26]); and model failures (e.g., switches or links that are known to fail with some probability [10]).

However, the semantics of ProbNetKAT is surprisingly subtle. Using the iteration operator (i.e., the Kleene star from regular expressions), it is possible to write programs that generate continuous distributions over an uncountable space of packet history sets [8, Theorem 3]. This makes reasoning about convergence non-trivial, and requires representing infinitary objects compactly

*This is a preliminary draft from March 21, 2018

[Rejected!]

A language for **modeling & reasoning** about networks **probabilistically.**



A language for modeling & reasoning about networks probabilistically.

$$[p] \in 2^{Pkt} \rightarrow Dist(2^{Pkt})$$

[p] ∈
$$2^{Pkt} \rightarrow 2^{Pkt}$$

Net + KA + T

network regular boolean primitives expressions tests

probabilistic primitives

 $p \oplus_r q$

network

f:=n, dup

expressions

&, ·, *

f=n

Verify reachability properties

Verify reachability properties

♦ but for probabilistic networks

Verify reachability properties

♦ but for probabilistic networks

Verify fault tolerance

Verify reachability properties

but for probabilistic networks

Verify fault tolerance

★ k-resilience

Verify reachability properties

but for probabilistic networks

Verify fault tolerance

- ★ k-resilience
- ◆ "is scheme A is more resilient than scheme B?"

Verify reachability properties

but for probabilistic networks

Verify fault tolerance

- ★ k-resilience
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- probability of packet delivery

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- ♦ k-resilience
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Compute quantitative network metrics

Verify reachability properties

but for probabilistic networks

Verify fault tolerance

- ♦ k-resilience
- ◆ "is scheme A is more resilient than scheme B?"
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Compute quantitative network metrics

◆ "expected number of hops?"

Verify reachability properties

♦ but for probabilistic networks

Verify fault tolerance

- ♦ k-resilience
- ◆ "is scheme A is more resilient than scheme B?"
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Compute quantitative network metrics

- ◆ "expected number of hops?"
- ◆ "expected link congestion?"

Verify reachability properties

♦ but for probabilistic networks

Verify fault tolerance

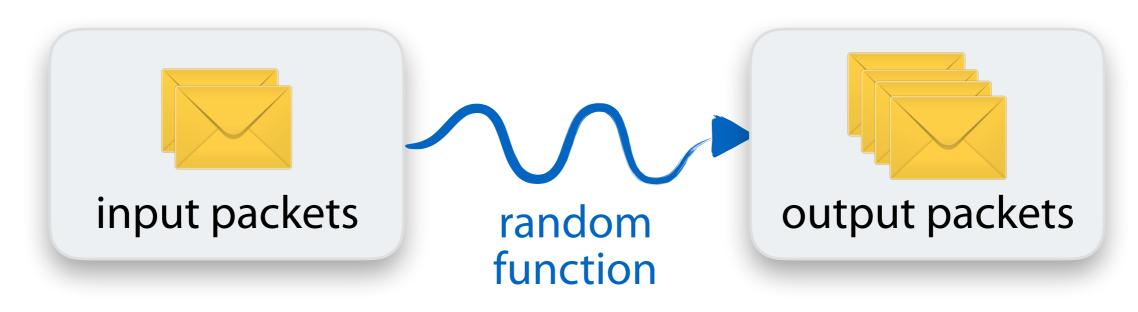
- ♦ k-resilience
- ◆ "is scheme A is more resilient than scheme B?"
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Compute quantitative network metrics

- ◆ "expected number of hops?"
- ◆ "expected link congestion?"
- ◆ computes analytical solution, not approximation

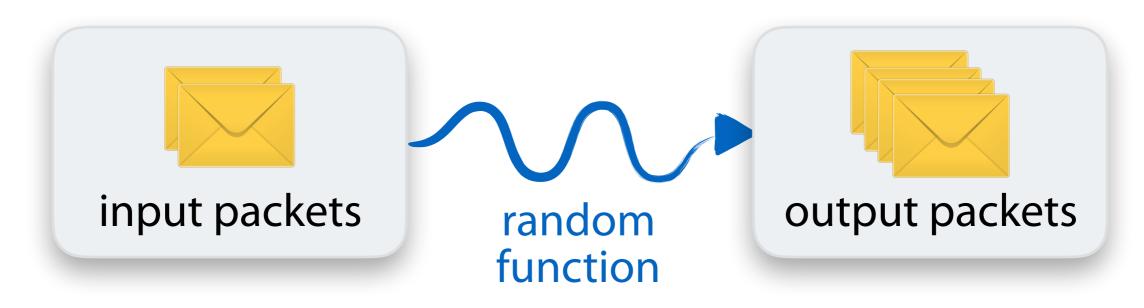
Semantics, Intuitively

Programs are **random** packet processing functions:



Semantics, Intuitively

Programs are **random** packet processing functions:



They can be modeled as Markov chains:

- states are given by sets of packet: $S = 2^{Pk}$
- Chain is given by transition matrix $\mathbf{B} \in [0,1]^{S\times S}$
- B(a, b): probability of producing output b on input a
- state space large, but finite!

Semantics, Formally

"Big Step" Semantics **B[p]**

- captures input-output behavior of p
- finitely representable & explicitly computable
- to decide p = q, simply check B[p] = B[q]
- challenge: how to define & compute B[[p*]]?

"Small Step" Semantics **S[p]**

- models single iteration of p*
- B[[p*]] defined as n step behavior of S[[p]], for $n \to \infty$
- limit can be computed explicitly, using theory of absorbing Markov chains!

Big Step Semantics

For deterministic primitives, **B** is just a 0-1-matrix.

$$\mathbf{B[[false]]} := \begin{bmatrix} \emptyset & b_2 \cdots & b_n \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_n & 1 & 0 & \cdots & 0 \end{bmatrix}$$

Similarly for $p \in \{ true, f=n, f:=n \}$.

Big Step Semantics

Program operators translate to matrix operations.

$$\mathbf{B} \llbracket \mathbf{p} \cdot \mathbf{q} \rrbracket := \begin{bmatrix} b_1 & \cdots & b_n & \cdots & c & \cdots \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 1/3 & 1/3 & 1/3 & \end{bmatrix} \bullet \begin{bmatrix} 1/6 & \vdots & b_1 \\ \vdots & 2/6 & \vdots & \vdots \\ 3/6 & \end{bmatrix} b_n$$

$$\mathbf{B} \llbracket \mathbf{p} \rrbracket \qquad \mathbf{B} \llbracket \mathbf{q} \rrbracket$$

Similarly for p & q, p $\oplus_{\mathbf{r}}$ q.

Problem: Kleene Star

Question

Let π_1 ! be the program that generates packet π_1 Consider p = (true $\oplus \pi_1$!). What is **B**[[p*]]($\{\pi_0\}$, $\{\pi_0\}$)?

Answer

$$\mathbf{B}[[p^{(n)}]](\{\pi_0\}, \{\pi_0\}) = (1/2)^n$$

Thus,
$$B[p^*](\{\pi_0\}, \{\pi_0\})$$

$$= \lim_{n \to \infty} B[p^{(n)}](\{\pi_0\}, \{\pi_0\})$$

$$= \lim_{n \to \infty} (1/2)^n = 0$$

But how to compute these limits in general?

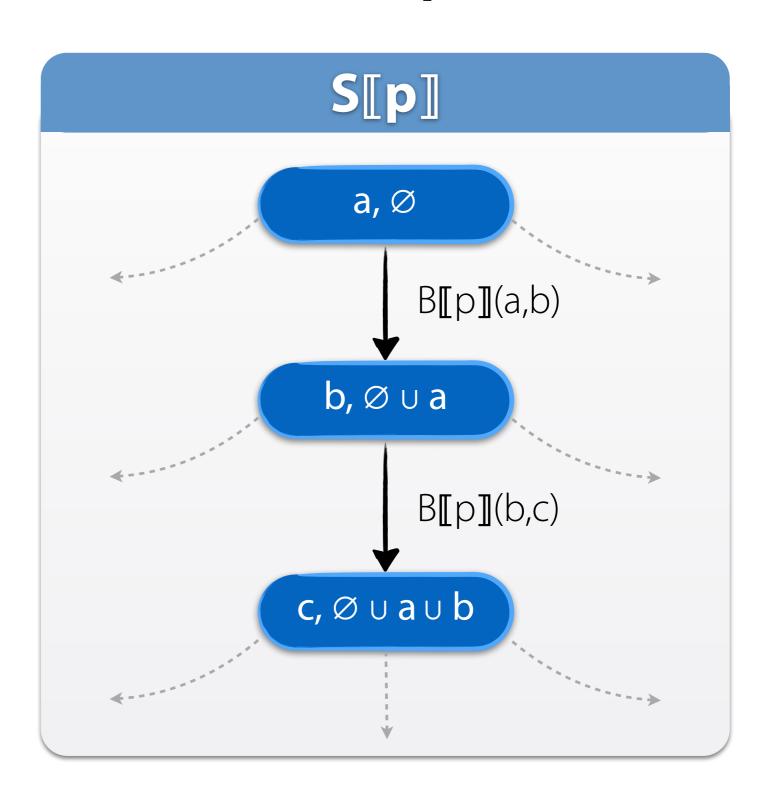
Small Step Semantics

1 step in $S[p] \approx 1$ iteration of p^*

In one iteration, p*:

- executes p to get new set of packets
- emits previous setof packets

$$B\llbracket p^*\rrbracket := \lim_{n\to\infty} S\llbracket p\rrbracket^n$$



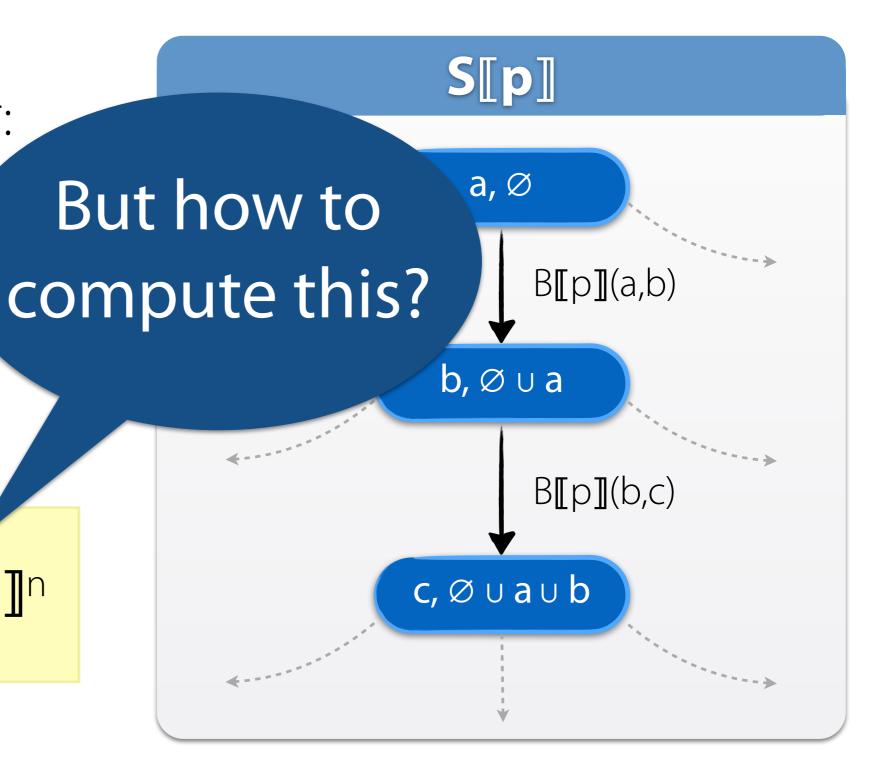
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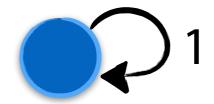
 $B[p^*] := \lim_{n \to \infty} S[p]^n$



Absorbing Markov Chains

S[p] can be "massaged" into an absorbing Markov chain

Absorbing state: 1



Absorbing chain: any state can reach absorbing state

Crucial property: for #steps $\rightarrow \infty$, will reach absorbing state with probability 1 (no matter the start state)

$$T = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}$$

$$\lim_{n \to \infty} T^n = \begin{bmatrix} I & 0 \\ (I - Q)^{-1} R & 0 \end{bmatrix}$$

Absorbing Markov Chains

S[p] can be "massaged" into an absorbing Markov chain

Absorbing sta

Absorbing

Crucial pro

state with prob

So $B[p^*] = \lim_{n \to \infty} S[p]^n$ can be computed explicitly!

rbing

tate

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F10: A Fault-Tolerant Engineered Network

Vincent Liu, Daniel Halperin, Arvind Krishnamurthy, Thomas Anderson University of Washington [NSDI'13]

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Motivation

♦ short-term failures in data centers are common

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- ◆ short-term failures in data centers are common
- ◆ application performance suffers

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Motivation

- ◆ short-term failures in data centers are common
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Solution

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- short-term failures in data centers are common
- ◆ application performance suffers
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Solution

→ detect failures of neighboring links & switches...

F10: A Fault-Tolerant Engineered Network

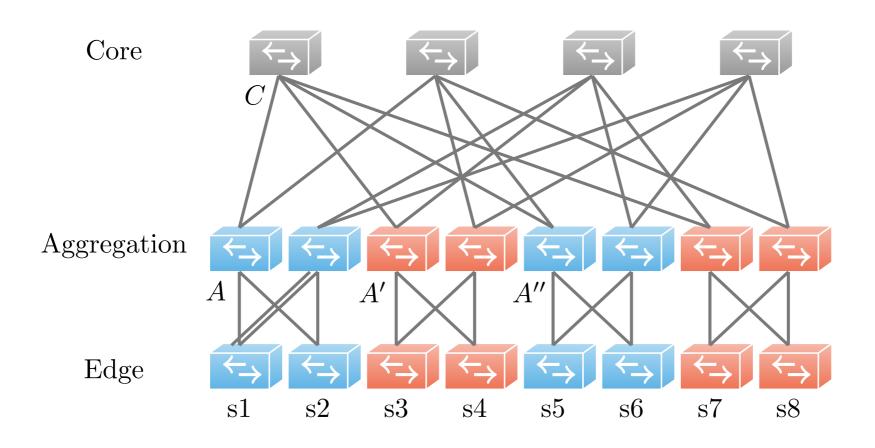
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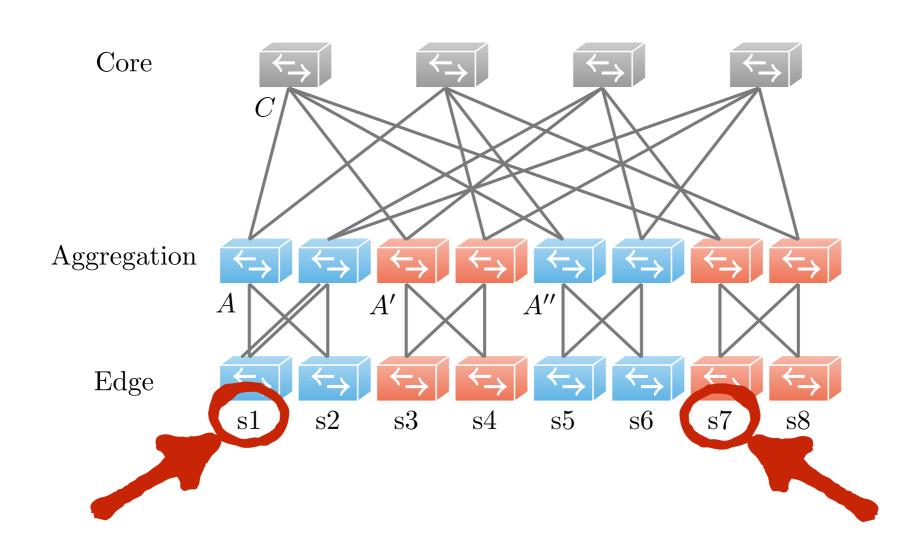
Motivation

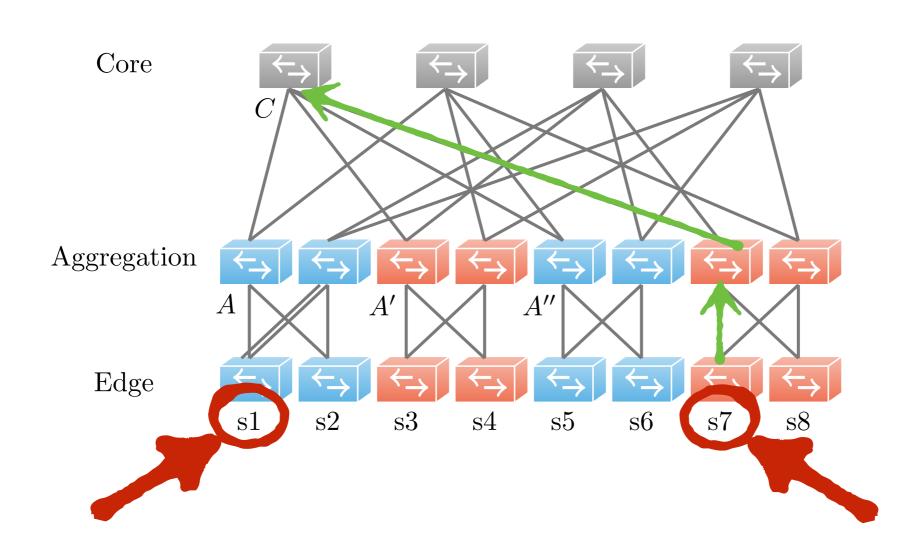
- short-term failures in data centers are common
- → application performance suffers
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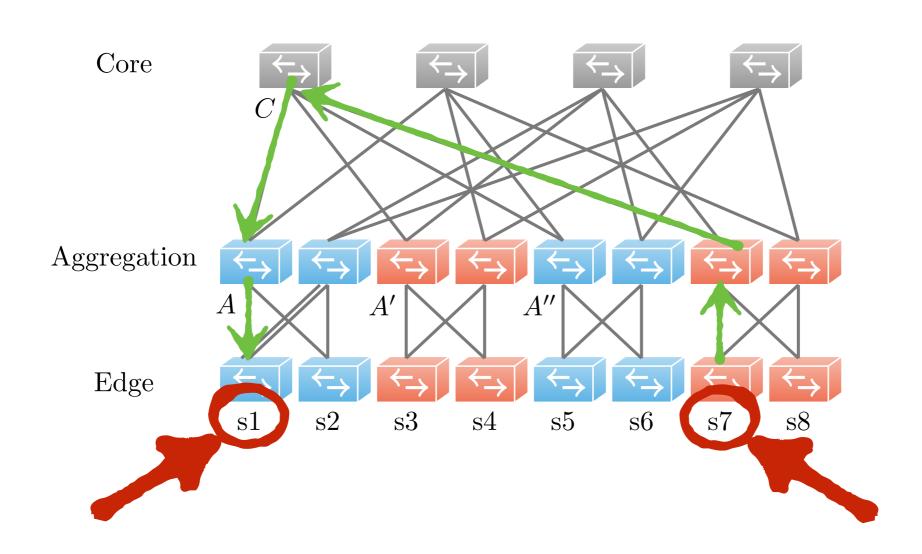
Solution

- → detect failures of neighboring links & switches...
- → ...and route around them

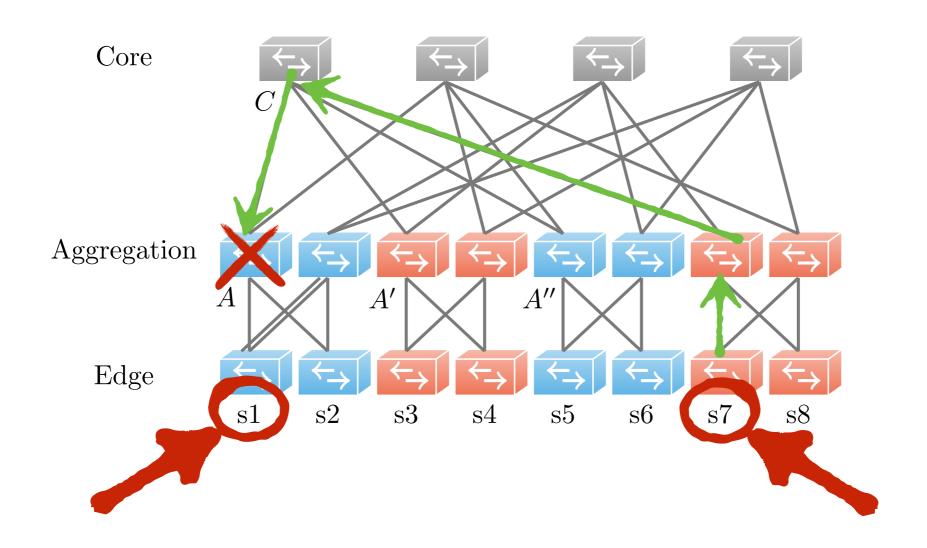






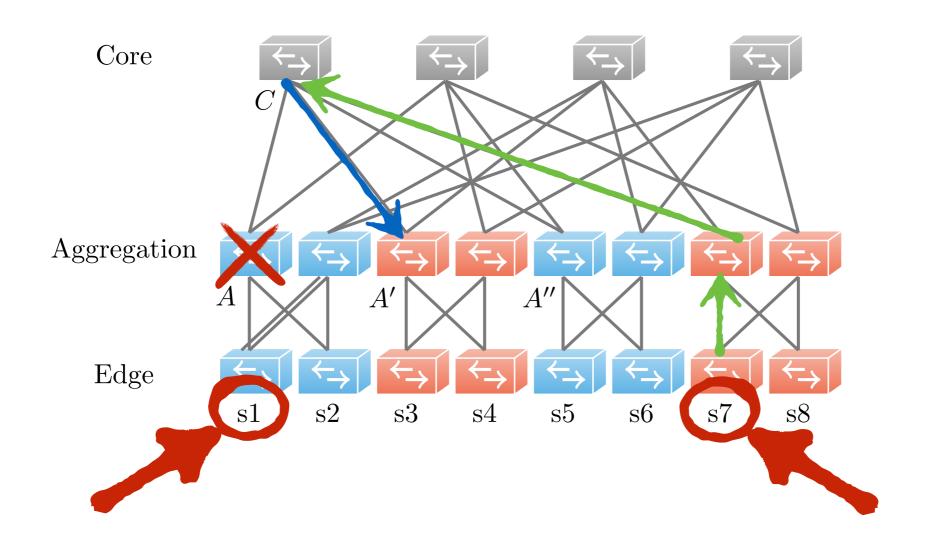


An ABFatTree is much like a regular FatTree



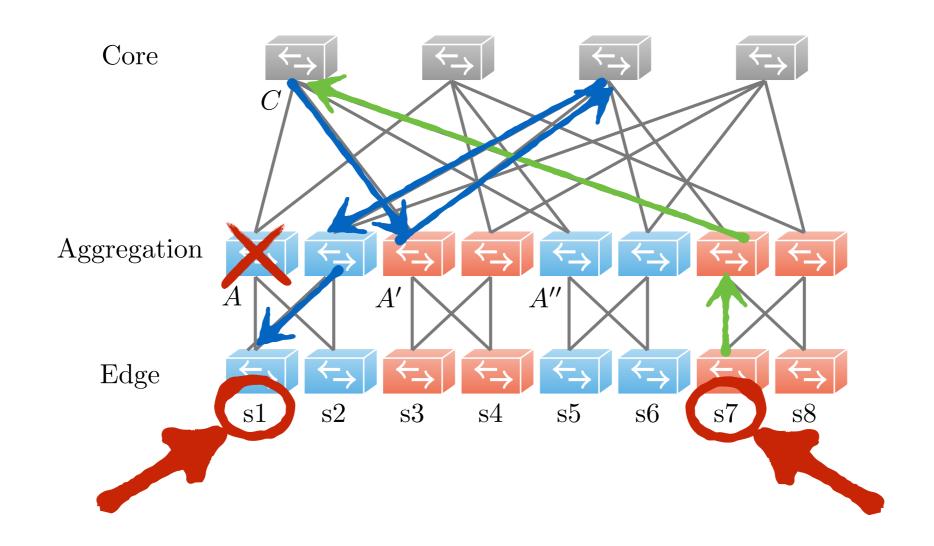
But it provides shorter detours around failures

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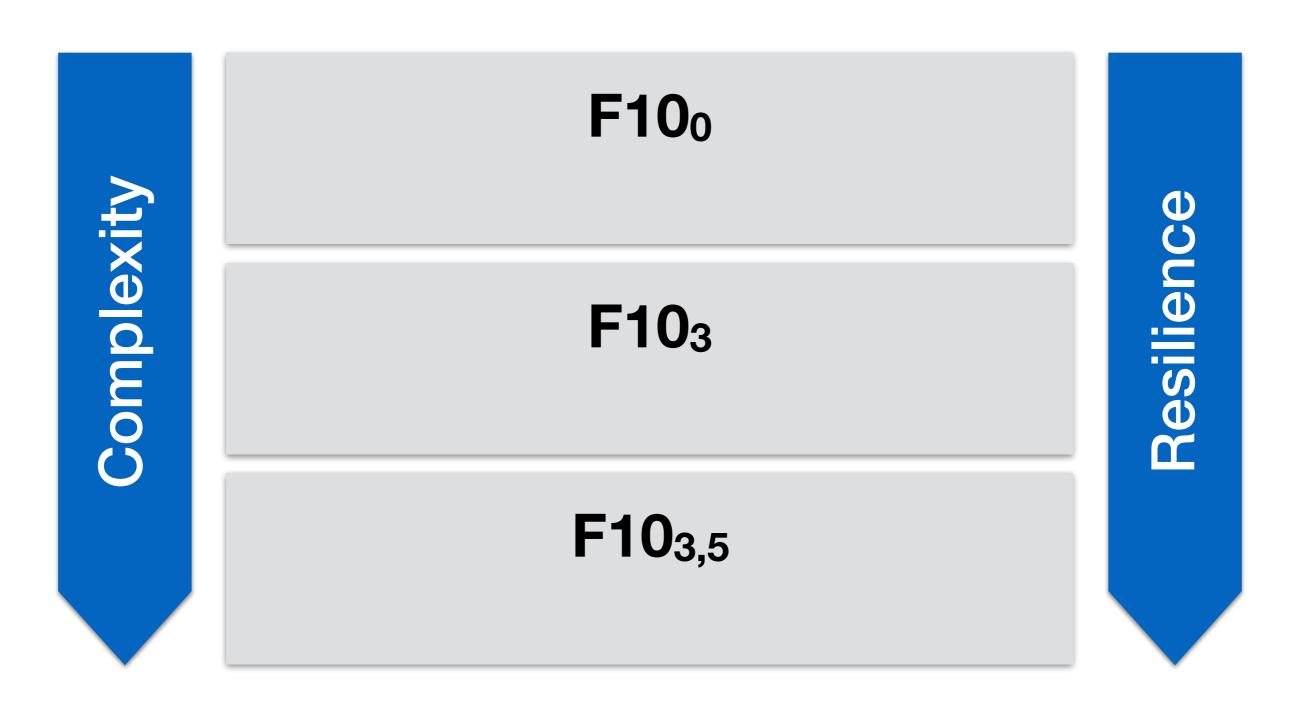
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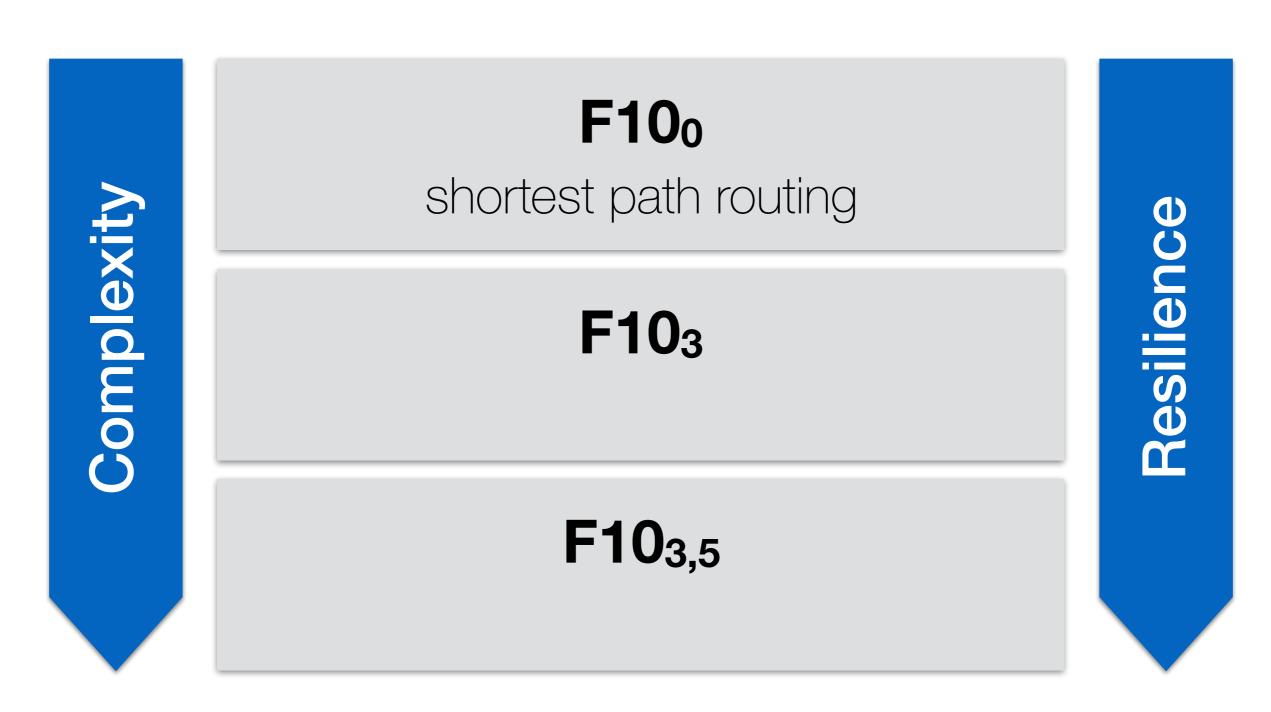
But it provides shorter detours around failures

We implemented F10 as a series of 3 refinements

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We implemented F10 as a series of 3 refinements



We implemented F10 as a series of 3 refinements



Resilience

We implemented F10 as a series of 3 refinements

Complexity

F10₀

shortest path routing

F10₃

 $F10_0 + 3$ -hop rerouting

F10_{3,5}

F10₃ + 5-hop rerouting

Resilience

We implemented F10 as a series of 3 refinements

Complexity

F10₀

shortest path routing

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F10_{3,5}

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oroutina

Resilience

Requires State

We verified k-resilience using ProbNetKAT

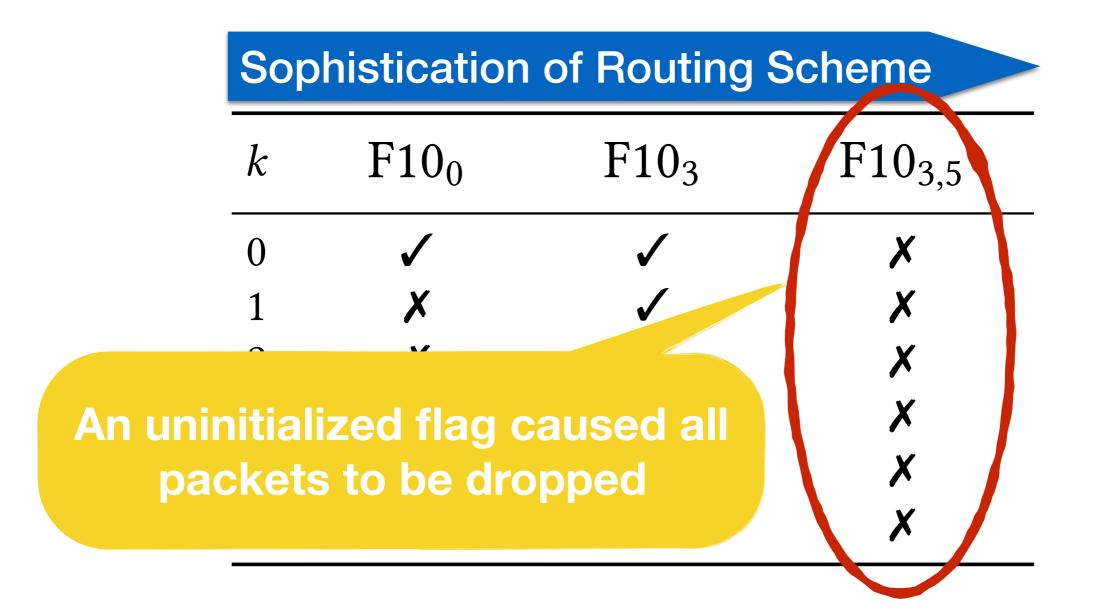
We verified k-resilience using ProbNetKAT

Sophistication of Routing Scheme					
\overline{k}	F10 ₀	F10 ₃	F10 _{3,5}		
0	√	✓	X		
1	X		×		
2	X		×		
3	X	X	×		
4	X	X	×		
∞	X	×	×		

We verified k-resilience using ProbNetKAT

Sophistication of Routing Scheme						
\overline{k}	F10 ₀	F10 ₃	F10 _{3,5}			
0	√	√	X			
1	X		X			
2	X		X			
3	X	X	X			
4	X	X	X			
	X	X	X			

We verified k-resilience using ProbNetKAT



k = number of failures

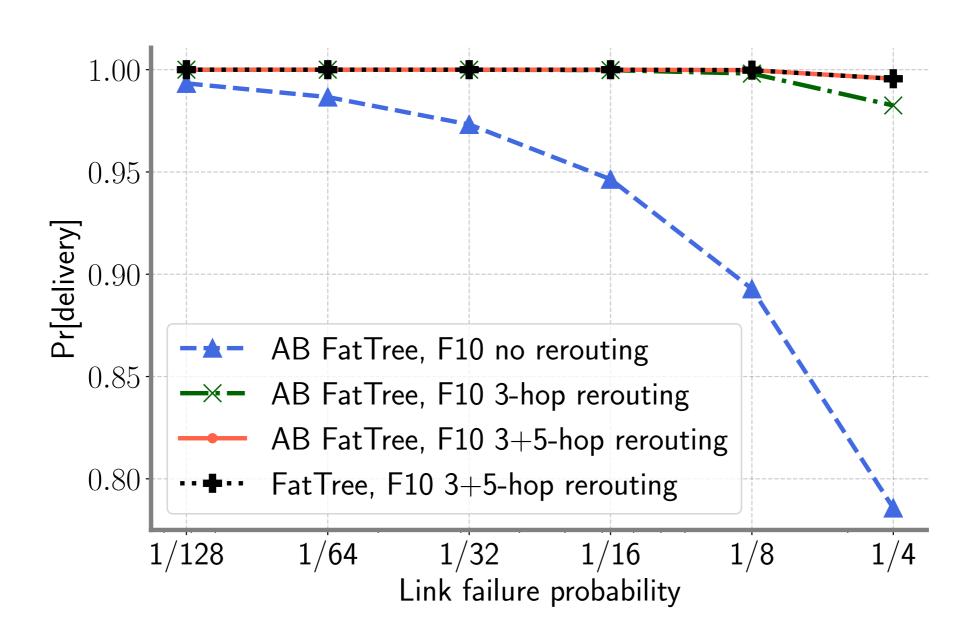
= 100% packet delivery

After fixing the bug...

Sophistication of Routing Scheme						
\overline{k}	F10 ₀	F10 ₃	F10 _{3,5}			
0	√	√	√			
1	X					
2	X					
3	X	X				
4	X	X	X			
∞	X	X	X			

Case Study: probability of delivery

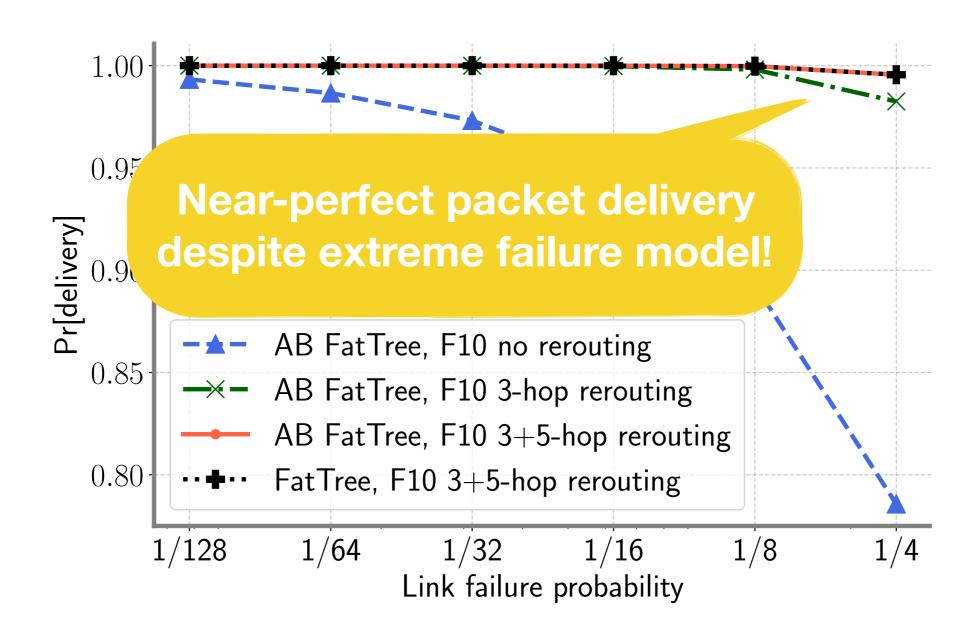
We evaluated packet loss when link failures increase



Dramatic improvement when using rerouting

Case Study: probability of delivery

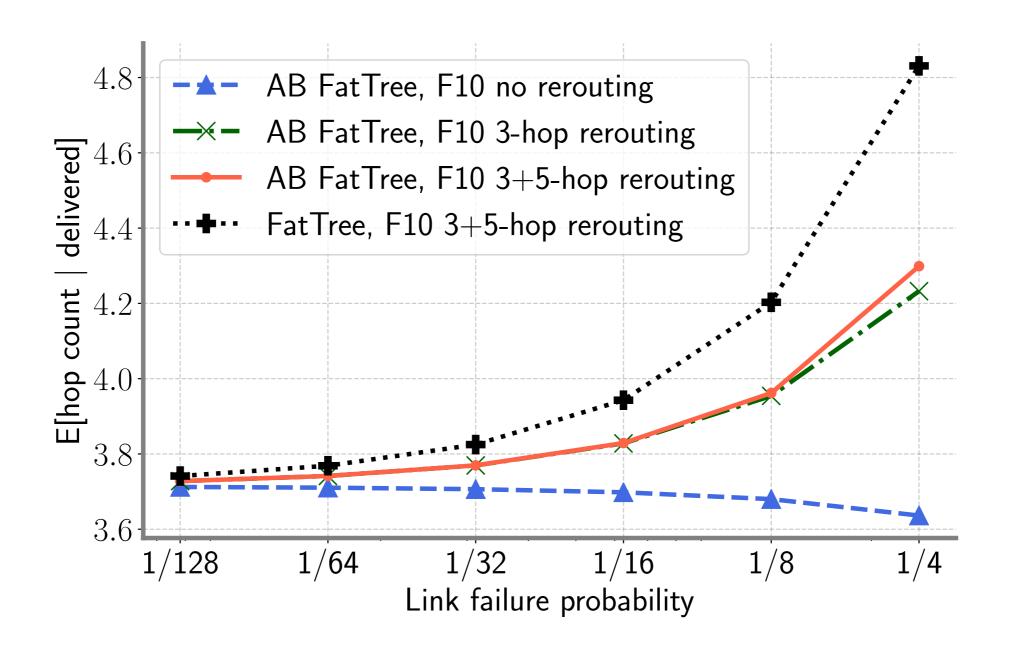
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Dramatic improvement when using rerouting

Case Study: expected hop count

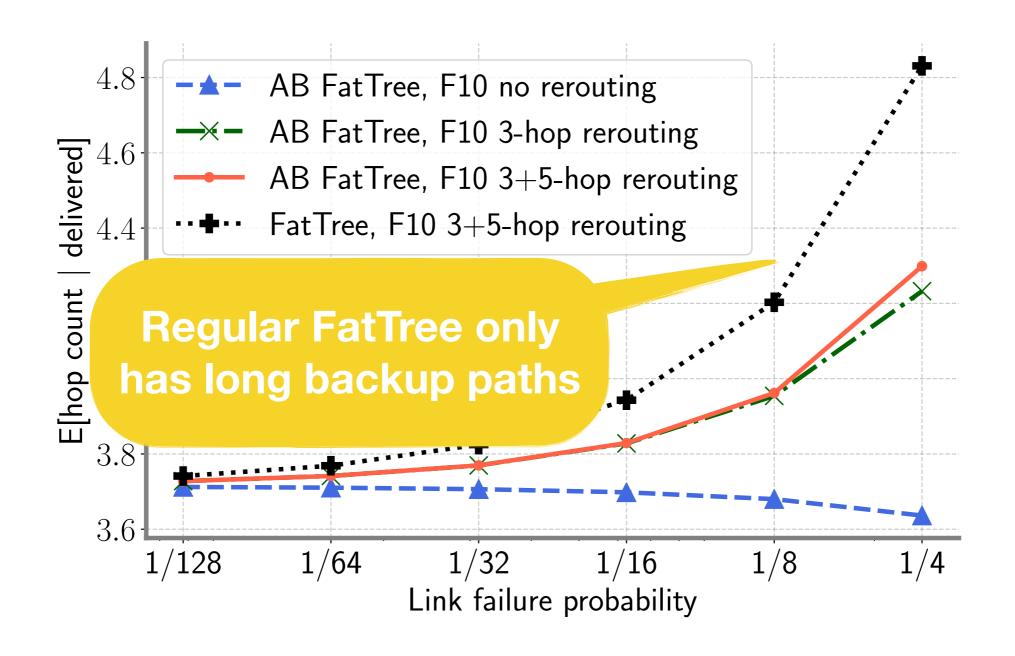
The price of resilience: increased paths lengths



ABFatTree outperforms regular FatTree

Case Study: expected hop count

The price of resilience: increased paths lengths



ABFatTree outperforms regular FatTree

Wrapping Up

ProbNetKAT is the first tool for specifying and verifying probabilistic networks

ProbNetKAT is the first tool for specifying and verifying **probabilistic** networks

Can verify reachability properties even if network behavior is not deterministic

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Can verify reachability properties even if network behavior is not deterministic

Can reason about resilience

e.g., k-resilience, probability of delivery

ProbNetKAT is the first tool for specifying and verifying **probabilistic** networks

Can verify reachability properties even if network behavior is not deterministic

Can reason about resilience e.g., k-resilience, probability of delivery

Can reason about quantitative properties e.g., expected path length under failure model

Scalable implementation

Heuristics that enable representing sparse matrices efficiently

Scalable implementation

Heuristics that enable representing sparse matrices efficiently

Probabilistic Inference

Given observation of packet loss, what link failure is most likely to have occurred?

Scalable implementation

Heuristics that enable representing sparse matrices efficiently

Probabilistic Inference

Given observation of packet loss, what link failure is most likely to have occurred?

Language Design

ProbNetKAT has no notion of queueing or time

ROBABILISTIC